Variational Analysis, PDEs and Mathematical Economics

BOOKLET OF ABSTRACTS

September, 19-20, 2019

Messina, Italia

Variational Analysis, PDEs and Mathematical Economics

In the occasion of Antonino Maugeri's 75th birthday

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On the relations between principal eigenvalue and torsional rigidity

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The relations between principal eigenvalue of the Laplace operator and torsional rigidity are studied in the class of general domains, convex domains, and domains with a small thickness. This is of help to provide some bounds for the Blasche-Santaló diagram of the two quantities

Propagation of singularities for solutions to Hamilton-Jacobi equations

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The study of the structural properties of the set of points at which the viscosity solution of a first order Hamilton-Jacobi equation fails to be differentiable - in short, the singular set - started with the paper [On the singularities of viscosity solutions to Hamilton-Jacobi-Bellman equations, Indiana Univ. Math. J. 36 (1987), pp.501–524] by Mete Soner and myself. These thirty years have registered enormous progress in the comprehension of the way how singularities propagate: we can now provide a fine topological analysis of the singular set, describe singular dynamics by generalised characteristics, and even study the long time behaviour of singular curves. In my talk, I will revisit the milestones of this theory and discuss possible developments and open problems.

Sliding modes for a system of PDEs controlling tumor growth

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In this talk, a tumor growth model, which leads to a viscous Cahn-Hilliard system for the phase variable coupled with a reaction-diffusion equation for the nutrient, will be considered. Then, we deal with the problem of controlling the system of partial differential equations and related conditions by a sliding mode (SM) type control. Well-posedness results and some regularity properties are obtained via variational analysis and hold for the state system, also including the state-feedback control law. Moreover, it turns out that the chosen SM control law forces the system to reach within finite time the sliding manifold, which is fixed in order that the tumor phase remains constant in time.

Ghost penalties in nonconvex constrained optimization: Diminishing stepsizes and iteration complexity

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We consider nonconvex constrained optimization problems and propose a new approach to the convergence analysis based on penalty functions. We make use of classical penalty functions in an unconventional way, in that penalty functions only enter in the theoretical analysis of convergence while the algorithm itself is penalty-free. Based on this idea, we are able to establish several new results, including the first general analysis for diminishing stepsize methods in nonconvex, constrained optimization, showing convergence to generalized stationary points, and the first complexity study for SQP-type algorithms.

Hints for Visiting again Pontryagin Maximum Principle and Extensions

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Image Space Analysis (ISA) for Constrained Extremum Problems (CEP) is shortly recalled. Unlike CEP in a Euclidean space and those of isoperimetric type, the image space associated with CEP, like e.g. of geodesic or control type, is ”naturally” infinite dimensional. Instead, it is possible to maintain it finite dimensional also for these kinds of problems: the infinite dimensionality is then confined to a selection function from a point-to-set map, which, in the spirit of Lagrange multipliers, allows us to postpone its handling to the writing of the necessary condition. A straightforward consequence is an extension of the classic Lagrange multiplier (adjoint variable), which is factorized as the product of a selection function, depending on the unknown, and a scalar number, which turns out to be the derivative of the perturbation function of CEP. Through a simple, classic example, this approach is outlined for the case of optimal control, giving hints for visiting again Pontryagin Maximum Principle. Several topics of optimal control, like the handling of state constraints, regularity/normality/non-degeneracy, should receive a simplified analysis.

Elliptic and parabolic equations under general and p,q-growth conditions

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We consider *variational solutions* to the Cauchy-Dirichlet problem

$$\begin{cases} \partial_t u = divD_\xi f(x,u,Du) - D_u f(x,u,Du) & \text{in } \Omega_T \\ u = u_0 \quad \text{on } \partial_{par}\Omega_T \end{cases}$$

where the function $f = f(x,u,\xi)$, $f : \mathbb{R}^n \times \mathbb{R}^N \times \mathbb{R}^{N\times n} \to [0,\infty)$, is convex with respect to (u,ξ) and coercive in $\xi \in \mathbb{R}^{N\times n}$, *but f not necessarily satisfies a growth condition from above*. A motivation to consider a class of such energy functions f can be also easily found in the stationary case, where a large literature in the *calculus of variations* is devoted to the minimization of *general* and $p,q-$*growth problems*. In the parabolic context the notion of variational solution, introduced by Bögelein-Duzaar-Marcellini, is compatible with the lack of *the same polynomial growth* from below and from above.

A journey through Mathematical Analysis and Optimization between the Two Sicilies: half a century of faces and ideas

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We present men and women, who with their ideas made half a century history in some branches of Mathematics in the south of Italy. The achieved results are evidenced by the ranking related to Mathematics that appears in the Ranking ARWU Mathematics 2019.

L^p-Theory of Venttsel BVPs with Discontinuous Data

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We will present some very recent results regarding the regularity and solvability theory of second-order elliptic equations with discontinuous coefficients coupled with boundary conditions of Venttsel type, given in terms of second-order differential operators with discontinuous data. Precisely, we deal with the problem

$$\begin{cases} \mathcal{L}u := -a^{ij}(x)D_iD_ju + b^i(x)D_iu + c(x)u = f(x), \\ \mathcal{B}u := -\alpha^{ij}(x)d_id_ju + \beta^i(x)d_iu + \beta^0(x)\partial_{\mathbf{n}}u + \gamma(x)u = g(x) \end{cases}$$

over a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$, with $C^{1,1}$-smooth boundary. Here $\mathbf{n}$ is the unit outward normal to $\partial\Omega$ and du stands for the tangential gradient of u, $d_i = D_i - \mathbf{n}_i\mathbf{n}_jD_j$. The solutions are understood in a *strong* sense, that is, these belong to suitable Sobolev spaces of twice weakly differentiable functions and satisfy the equations above at *almost every* point x. The natural space $V_{p,q}(\Omega)$ to study that problem consists of all functions $u \in W_p^2(\Omega)$ with traces in $W_q^2(\partial\Omega)$, where the exponents p and q satisfy $1 < p \leq \frac{nq}{n-1} < p^*$, $q > 1$, with p^* being the Sobolev conjugate of p. The principal coefficients a^{ij} and α^{ij} of the *uniformly elliptic* operators $\mathcal{L}$ and $\mathcal{B}$ are supposed to be *VMO*-functions in Ω and $\partial\Omega$, respectively, while optimal Lebesgue or Orlicz integrability is required on the lower-order coefficients.
We derive, first of all, an *a priori* estimate for the $V_{p,q}(\Omega)$-norm of any strong solution in terms of $\|f\|_{L^p(\Omega)}$ and $\|g\|_{L^q(\partial\Omega)}$. Based on this, the *elliptic regularization property* is obtained for the couple $(\mathcal{L}, \mathcal{B})$ in the framework of the Sobolev spaces and, under additional assumptions on the vector field $\left(\beta^1, \ldots, \beta^n\right)$ and the coefficients c and γ, *strong solvability* is proved for the Venttsel BVP in $V_{p,q}(\Omega)$ for *all* admissible values of p and q.
Strong solvability of the *quasilinear* Venttsel problem

$$\begin{cases} -a^{ij}(x,u)D_iD_ju + a(x,u,Du) = 0 & \text{a.e. in } \Omega, \\ -\alpha^{ij}(x,u)d_id_ju + \alpha(x,u,Du) = 0 & \text{a.e. on } \partial\Omega \end{cases}$$

with *discontinuous* coefficients will be discussed as well.
The results are obtained in collaboration with Alexander Nazarov (St. Petersburg), Darya Apushkinskaya (Saarbrücken) and Lubomira Softova (Salerno).

Biduality Theorems for Lipschitz– Holder spaces and other big–spaces

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Let (K, ρ) be a compact metric space and $\mathcal{M}(K) = \mathcal{M}(K, \rho)$ be the set of all finite Borel measures on K. The Lipschitz space Lip(K, ρ) is defined by the condition

$$\sup_{\substack{x,y \in K \\ x \neq y}} \frac{|u(x) - u(y)|}{\rho(x, y)} < \infty$$

and its closed subspace lip(K, ρ) by the vanishing property

$$\lim_{\rho(x,y) \to 0} \frac{|u(x) - u(y)|}{\rho(x, y)} = 0.$$

When $\mathcal{M}(K)$ is equipped with the usual total variation norm

$$\|\mu\| = |\mu|(K) \qquad \mu \in \mathcal{M}(K)$$

it is isometrically isomorphic to $(\mathcal{C}_0(K))^*$, but its dual $(\mathcal{M}(K))^*$ has no trasparent description.
On the other hand, supplying $\mathcal{M}(K, \rho)$ with another norm inventend by Kantorovich, the (KR)– norm $\|u\|_\rho$ one has

$$(\mathcal{M}(K, \rho))^* \simeq \text{Lip}(K, \rho)$$

isometrically, and $\mathcal{M}(K, \rho)$ enjoies atomic decomposition.
Choosing $K = Q_0 = [0, 1]^n$, $\rho_\alpha(x, y) = |x - y|^\alpha$, $0 < \alpha < 1$, it is classical that

$$(\text{lip}(K, \rho_\alpha))^{**} \simeq \text{Lip}(K, \rho_\alpha) \tag{1}$$

that is

$$\left(c^{0,\alpha}(Q_0)\right)^{**} \simeq \mathcal{C}^{0,\alpha}(Q_0)$$

isometrically.
On the other hand, for $\alpha = 1$, (1) fails.
These results are related to the duality theory of Banach Function Spaces recently developed in connection with Bourgain-Brezis-Mironescu paper.

Two non-linear generalizations of Neumann's Lemma

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In this talk I will present new invertibility theorems for non-necessary linear operators on a real Banach space, similar to the linear Cazassa-Christenses Lemma. Such invertibility results go back to the so-called Neumann Lemma.
This Lemma has various well-known consequences including two classical theorems:
- the set of all the invertible linear bounded operator defined on a real Banach space is open;
- the function which assigns to each linear bounded operator defined on a real Banach space its spectral radius is upper semi-continuous.
Our analysis benefits from the introduction of a new concept, strongly related to the Birkhoff-James orthogonality, that is the notion of near operator introduced at the end of the eighties by Sergio Campanato in a series of papers, through which he was able to unify existence and regularity results for PDE's and systems in non-divergence based on the Schauder method, the Lax-Milgram theorem, the Cordes theorem and the theory of monotone operators.
This talk summarizes a joint work with Emil Ernst and Annamaria Barbagallo.

Existence of financial equilibria with real assets using a Variational Inequality approach

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We present a two-period general equilibrium model with incomplete financial markets and real assets and analyze the existence problem by using a variational inequality approach. In the case of real assets, equilibria may not exist since changes in prices may change the rank of the return matrix causing a discontinuity in the demand function. The possibility of changes of rank of the return matrix was addressed by the seminal paper by Duffie and Shafer (1985), considering a fictitious definition of equilibrium, in which, the rank of the matrix is fixed. In the present paper, we present a different definition of fictitious equilibrium, which is shown to be equivalent to the definition of equilibrium under a rank condition. To the aim to show existence of fictitious equilibria, we derive a variational formulation of the problem and we provide an existence theorem for a generalized quasi-variational problem involving Grassmannian manifolds. We then introduce a sequence of those variational problems and we show that the associated sequence of solutions converges to the chosen definition of equilibrium.